WRITTEN HOMEWORK #6, DUE FEB 26, 2010

(1) (Chapter 17.5, #21, 22)
(a) Show that any vector field of the form

$$\mathbf{F}(x, y, z) = \langle f(x), g(y), h(z) \rangle$$

is irrotational. (This means that $\nabla \times \mathbf{F} = \mathbf{0}$.)
(b) Show that any vector field of the form

$$\mathbf{F}(x, y, z) = \langle f(y, z), g(x, z), h(x, y) \rangle$$

is incompressible. (This means that $\nabla \cdot \mathbf{F} = 0$.)

(2) (Chapter 17.6, #38) Find the area of the part of the plane $2x + 5y + z = 10$ which satisfies $x^2 + y^2 \leq 9$.

(3) (Chapter 17.6, #46) Find the area of the helicoid (spiral ramp) with parameterization

$$\mathbf{r}(u, v) = \langle u \cos v, u \sin v, v \rangle, 0 \leq u \leq 1, 0 \leq v \leq \pi.$$

(For a picture of this surface, see figure IV of page 1115 in the text.)

(4) (Chapter 17.7, #20) Let $\mathbf{F} = \langle y, x, z^2 \rangle$. If S is the helicoid from the previous problem, find the surface integral of $\mathbf{F}$ over S. (The orientation is the one which points in the same direction as the fundamental vector product).

(5) (Chapter 17.7, #22) Let $\mathbf{F} = \langle x, y, z^4 \rangle$. Let S be the part of the cone $z = \sqrt{x^2 + y^2}$ which lies beneath $z = 1$, with downwards pointing orientation. Find the surface integral of $\mathbf{F}$ over S.

(6) (Chapter 17.7, #27) Let $\mathbf{F} = \langle x, 2y, 3z \rangle$, and let S be the cube with vertices $(\pm 1, \pm 1, \pm 1)$, with outward orientation. Find the surface integral of $\mathbf{F}$ across S.